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Effect of Upstream Conditions on the Outer Flow of Turbulent Boundary Layers

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It will be shown that the boundary layer develops differently depending on the upstream conditions (e.g., the wind-tunnel speed, size of tripping wire, or recent history). Also, it will be demonstrated that almost all of the Reynolds-number dependence observed in the outer velocity deficit profiles is caused primarily by changes in the upstream conditions and not to the local Reynolds number. The empirical velocity scale of Zagarola and Smits (Zagarola, M. V., and Smits, A. J., "Mean-Flow Scaling of Turbulent Pipe Flow," *Journal of Fluid Mechanics*, Vol. 373, 1998, pp. 33–79) is derived here for boundary layers with and without pressure gradient using similarity principles. This scaling is successful in removing the effect of upstream conditions and the residual dependence on the local Reynolds number from the mean velocity deficit. Even more interesting, it produces only three profiles in turbulent boundary layers, regardless of the strength of the pressure gradient: one for adverse pressure gradient, one for favorable pressure gradient, and one for zero pressure gradient. These results are consistent with results of Castillo and George (Castillo, L., and George, W. K., "Boundary Layers with Pressure Gradient: Similarity of the Velocity Deficit Region," AIAA Paper 2000-0913, Jan. 2000) obtained by means of similarity analysis of the Reynolds-averaged Navier–Stokes equations for equilibrium flows.

Nomenclature

f_{op}	=	outer velocity profile (at finite δ^+)
$f_{op\infty}$	=	asymptotic outer velocity profile (as $\delta^+ \to \infty$)
R_{so}	=	outer Reynolds-stress scale
R_θ	=	Reynolds number based on θ
U_{so}	=	outer velocity scale
U_∞	=	freestream velocity
$\boldsymbol{U}_\infty$	=	freestream velocity outside boundary layer
$U_\infty - U$	=	velocity deficit
$U_\infty(\delta_*/\delta)$	=	Zagarola/Smits velocity scale
u_*	=	friction velocity, $u_* = \surd(\tau_w/\rho)$
$\bar{y}$	=	y/δ
δ	=	boundary-layer thickness, for example, δ_{99}
δ_*	=	displacement thickness $\int_0^\infty \left(1 - \frac{U}{U_\infty}\right) \mathrm{d}y$
δ^+	=	$\delta u_*/\nu$
θ	=	momentum thickness $\int_0^\infty \left(\frac{U}{U_\infty}\right)\left(1 - \frac{U}{U_\infty}\right) \mathrm{d}y$
Λ	=	pressure parameter, $\Lambda \equiv (\delta/\rho U_\infty^2\, \mathrm{d}\delta/\mathrm{d}x)(\mathrm{d}P_\infty/\mathrm{d}x)$
$*$	=	(unknown) dependence on upstream conditions

I. Introduction

FOR more than 50 years researchers in the turbulence community have been trying to collapse mean velocity data for turbulent boundary layers, especially those with mean streamwise pressure gradient (for example, Refs. 1–7). The fact that these efforts have continued to the present makes it clear that such attempts have never been quite successful.

The most popular approach until recently was originally from von Kármán,[8] who suggested (on empirical grounds) a velocity deficit normalized by the friction velocity, that is, $(U_\infty - U)/u_*$. Clauser[5] used this empirical scaling together with the approach of Millikan[9] to deduce the universal log layer for boundary layers with pressure gradient. Coles[4] carried this approach one step further by introducing a wake function and a π parameter to account for the outer part of the boundary layer. Despite the early success of these approaches, there were always some problems. First, the π parameter did not appear to be reaching a Reynolds-number independent value.[4,10,11] Second, directly measured values of the shear stress did not collapse the log layer profiles, whereas values of the shear stress chosen to collapse the profiles (the so-called Clauser method) did not satisfy the momentum integral equation.[12,13]

There have been a number of recent developments of great interest:

1) First, Castillo and George,[2] George and Castillo,[7] and Castillo[14] analyzed the boundary-layer Reynolds-averaged equations using an equilibrium similarity analysis. They showed that the mean velocity profiles could be Reynolds-number invariant in the limit of infinite Reynolds number only if in the same limit the outer velocity scale was proportional to the freestream velocity U_∞. They were not, however, able to completely collapse the data with just U_∞ and attributed this failure to finite Reynolds-number effects.

2) Second, Zagarola and Smits[1] found an empirical velocity scale $U_\infty\delta_*/\delta$, which did successfully collapse the data for the outer flow of the zero-pressure-gradient (ZPG) turbulent boundary layers. Subsequently, Wosnik and George[15] showed that this scaling was consistent with the theory of George and Castillo[13] for ZPG boundary layers.

The main goal of this paper is to show that the effects of the upstream conditions play an important role in the development of the flow downstream. In fact, the Reynolds-number dependence observed in the outer mean velocity profiles (when normalized by the freestream velocity) is primarily caused by the changes in the upstream conditions, and not caused by the local Reynolds-number variation. Also, it will be shown that the Zagarola/Smits scaling successfully removes both the effect of upstream conditions and the residual Reynolds-number dependence from the velocity deficit profiles (that is, $y/\delta > 0.1$). Moreover, there appear to be only three basic velocity profiles [one each for ZPG, adverse pressure gradient (APG) and favorable pressure gradient (FPG)], consistent with the similarity analysis of Castillo and George[2] for equilibrium flows (that is, those with $\Lambda = \text{constant}$).

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II. Similarity Analysis

The Equilibrium Similarity Analysis of Castillo and George[2] and George and Castillo[13] depend strictly on the Reynolds-averaged equations of motion. The basic ideas and some of the most important results are shown next. These similarity ideas will be used later in the paper to determine the outer velocity scale proposed by Zagarola and Smits.[1]

A. Similarity Analysis

The outer mean velocity scale for turbulent boundary layers must be determined from an equilibrium similarity analysis of the governing equations and not chosen a priori or using dimensional analysis. George and Castillo[13] applied this concept to the Reynolds-averaged Navier–Stokes (RANS) equations for ZPG in order to determine the mean velocity and Reynolds shear-stress scales in the outer turbulent boundary layer as U_∞ and $U_\infty^2\,\mathrm{d}\delta/\mathrm{d}x \sim u_*^2$, respectively. These results also apply for pressure gradient (PG) turbulent boundary layers.

B. Similarity Solution Form

The basic assumption is that it is possible to express any dependent variable, in this case the outer mean velocity deficit $U - U_\infty$ and the Reynolds shear stress, as a product of two functions:

$$U - U_\infty = U_{\mathrm{so}}(x) f_{\mathrm{op}}(\bar{y}, \delta^+; \Lambda; *) \tag{1}$$

$$-\langle uv\rangle = R_{\mathrm{so}}(x) r_{\mathrm{op}}(\bar{y}, \delta^+; \Lambda; *) \tag{2}$$

where U_{so} and R_{so} depend on x only and need to be determined from the equations of motion. The arguments inside the similarity functions f_{op} and r_{op} represent the outer similarity coordinate $\bar{y} = y/\delta_{99}$, the Reynolds-number dependence $\delta^+ = \delta u_*/\nu$, the pressure gradient parameter Λ, and any possible dependence on the upstream conditions $*$, respectively. The upstream parameter $*$ is used in this paper to illustrate that the downstream flow may have such dependence. Some of the upstream parameters may be, but are not limited to, the freestream velocity at the tripping device, the turbulence intensity, or the geometry and location of the tripping device. Other possible upstream conditions could exist, such as suction or blowing, etc. The pressure gradient parameter Λ was determined via similarity analysis using the RANS equations and is given here as

$$\Lambda \equiv -\delta \Big/ \frac{U_\infty\,\mathrm{d}\delta}{\mathrm{d}x}\frac{\mathrm{d}U_\infty}{\mathrm{d}x} = \text{constant} \tag{3}$$

or equivalently

$$\Lambda \equiv \delta \Big/ \frac{\rho U_\infty^2\,\mathrm{d}\delta}{\mathrm{d}x}\frac{\mathrm{d}P_\infty}{\mathrm{d}x} = \text{constant} \tag{4}$$

C. Asymptotic Invariance Principle (AIP)

This principle is based on the fact that in the limit as the Reynolds number becomes infinite the outer boundary-layer equations become independent of Reynolds number. Because in the infinite local Reynolds-number limit the outer equations become independent of δ^+, so must properly scaled solutions to them. Hence, in this limit Eqs. (1) and (2) must also become independent of Reynolds number, that is,

$$f_{\mathrm{op}}(\bar{y}, \delta^+; \Lambda; *) \to f_{\mathrm{op}\infty}(\bar{y}; \Lambda, *) \tag{5}$$

$$r_{\mathrm{op}}(\bar{y}, \delta^+; \Lambda; *) \to r_{o\infty}(\bar{y}; \Lambda, *) \tag{6}$$

as $\delta^+ \to \infty$. The subscript op∞ is used to distinguish these infinite Reynolds-number solutions from the finite Reynolds-number profiles used before [Eqs. (1) and (2)]. It will be seen later that when ZPG and PG boundary layers are normalized by the Zagarola/Smits scaling $U_\infty\delta_*/\delta$, the Reynolds-number dependence and effects of the upstream conditions* vanish from the outer deficit profiles.

D. Outer Boundary-Layer Equations

The equations of motion and boundary conditions for the outer part $(y/\delta_{99} > 0.1)$ of a pressure gradient turbulent boundary layer (with constant properties) at high Reynolds number are well known to be given by the following[16]:

x momentum:

$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\mathrm{d}P_\infty}{\mathrm{d}x} + \frac{\partial}{\partial y}[-\langle uv\rangle] + \frac{\partial}{\partial x}[\langle v^2\rangle - \langle u^2\rangle] \tag{7}$$

continuity:

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \tag{8}$$

where $U \to U_\infty$ and $-\langle uv\rangle \to 0$ as $y \to \infty$. The viscous terms in the momentum equation have been neglected because they are small away from the near-wall region. Also, the last terms on the right-hand side are of second order in the turbulence intensity and are usually neglected.

E. Similarity Conditions

The scales for the outer velocity and Reynolds shear stresses are determined by substituting Eqs. (5) and (6) into the outer equations. Castillo and George[2] show that similarity is possible in the outer boundary layer only if $\Lambda = $ constant and

$$U_{\mathrm{so}} = U_\infty \tag{9}$$

$$R_{\mathrm{so}} = \frac{U_\infty^2\,\mathrm{d}\delta}{\mathrm{d}x} \tag{10}$$

They defined such boundary layers to be "equilibrium boundary layers," borrowing the term from Clauser.

For nonzero values of Λ, Eq. (3) can be integrated to yield

$$\delta \sim U_\infty^{-1/\Lambda} \tag{11}$$

Therefore, if an equilibrium flow exists at all, the data of $\log(U_\infty)$ vs $\log(\delta)$ must show a linear relationship with a constant slope $-1/\Lambda$. Surprisingly, Castillo and George[2] were able to show that most pressure gradient boundary layers are indeed equilibrium boundary layers and that the exceptions are nonequilibrium. The theory was shown to be valid even for flows near separation. In addition, they showed that only three values of Λ exist for turbulent boundary layers: one for APG with $\Lambda = 0.22$, one for FPG with $\Lambda = -1.915$, and one for ZPG with $\Lambda = 0$.

III. ZS Scaling

Using the similarity ideas outlined in the preceding section, the empirical scaling of Zagorola and Smits[1] $U_{\mathrm{so}} = U_\infty(\delta_*/\delta)$ will be derived in this section. It will be assumed that the function f_{op} can be expressed as a product of two functions, that is,

$$f_{\mathrm{op}}(\bar{y}, \delta^+; \Lambda; *) = G(\delta^+; *) F_{\mathrm{op}\infty}(\bar{y}; \Lambda) \tag{12}$$

The first, $G(\delta^+; *)$, contains the dependence on Reynolds number and the upstream conditions, whereas the second, $F_{\mathrm{op}\infty}(\bar{y}; \Lambda)$, contains the normalized dependence on distance from the wall $\bar{y}$ and the pressure parameter Λ. This profile $F_{\mathrm{op}\infty}$ represents the asymptotic velocity profile in the limit as $Re \to \infty$ for fixed upstream conditions. It is this profile that must reduce to a similarity solution of the RANS equations as required by the AIP. Hence this asymptotic profile must be independent of Reynolds number, but its shape might be different for ZPG, FPG, and APG turbulent flows depending on the values of the pressure parameter Λ. A similar decomposition of the profile was used by Wosnik and George[15] for ZPG boundary layers.

For incompressible flow the displacement thickness is given by

$$[U_\infty\delta_*] = \int_0^\infty (U_\infty - U)\,\mathrm{d}y \tag{13}$$

The function $G(\delta^+, *)$ can be determined by substituting Eq. (12) into the defining equation for the displacement thickness equation. The result is

$$[\delta_*] \approx [\delta G]\int_0^\infty f_{\mathrm{op}\infty}(\bar{y}, \Lambda)\,\mathrm{d}\bar{y} \tag{14}$$

where a small contribution from the inner layer has been neglected. It is easy to show that Eq. (14) is exact in the limit of infinite Reynolds number.

Now because the integrand of Eq. (14) is in similarity variables, the integral can at most depend on Λ, which must itself be independent of x. Therefore, $G\delta$ and δ_* must have the same x dependence. It follows immediately that

$$G \propto (\delta_*/\delta) \tag{15}$$

The function G can be combined with Eq. (12) and Eq. (1) to yield the outer velocity scale of Zagarola/Smits (Z/S) $U_{so} = U_\infty(\delta_*/\delta)$. Note that the fact that the Zagarola/Smits scaling contains the Reynolds-number dependence term δ_*/δ means that the boundary layer is indeed Reynolds-number dependent, exactly as argued by Castillo and George[2] and George and Castillo.[13]

George and Castillo (GC)[13] showed that $\delta_*/\delta \to$ constant in the limit of infinite local Reynolds number, but argued that the constant might depend on the upstream conditions. This result follows immediately from the Asymptotic Invariance Principle, which requires that any properly scaled similarity function must be asymptotically independent of Reynolds number. Thus, in the limit as $\delta^+ \to \infty$, $G \to G_\infty(*)$ only. Therefore, if the proposed separation of f_{op} is valid, all of the effects of upstream conditions should be removed by the Z/S scaling. Or conversely, if the Z/S scaling proves successful, then the separation must be at least approximately valid in the limit as $\delta^+ \to \infty$.

Because $\delta_*/\delta \to$ constant as $\delta^+ \to \infty$, the Z/S scaling $U_\infty(\delta_*/\delta)$ reduces to the GC scaling, U_∞ in the same limit. Thus both the U_∞ and $U_\infty\delta_*/\delta$ scalings are consistent with the equilibrium similarity analysis. The latter, of course, also removes the upstream and local Reynolds-number effects, if the separation hypothesis of Eq. (12) is correct. This can be contrasted with the analysis of Clauser, which requires that $U_\infty\delta_*/\delta \sim u_*$. Obviously if this classical result is correct, there should be no difference between the Z/S-scaled profiles and those using u_*, contrary to the finding of Zagarola and Smits.[1]

IV. Results

The goals of this section are fourfold:

1) First, it is to show that the Reynolds-number dependence observed in the outer mean velocity profiles mostly comes from the upstream conditions.

2) Second, it is to show that when the upstream conditions are kept fixed the velocity profiles collapse with just the freestream velocity U_∞.

3) Third, it is to show that the Zagarola/Smits scaling successfully removes the Reynolds-number dependence of the outer flow.

4) Fourth, it is to demonstrate that there appear to be only three basic profiles in turbulent boundary layers, regardless of the strength of the pressure gradient.

A. ZPG Velocity Profiles

Figure 1 shows the outer mean velocity profiles of three different ZPG turbulent boundary layers of Smith and Walker,[17] Purtell et al.,[18] and Österlund[19] normalized by U_∞ and δ_{99}. The Smith and Walker profiles vary in Reynolds number based on momentum thickness from about $3 \times 10^3 \le R_\theta \le 48.3 \times 10^3$. The data from Purtell et al. have a variation in Reynolds number from about $4.6 \times 10^2 \le R_\theta \le 5 \times 10^3$. Finally, the very recent data of Österlund vary in Reynolds number from about $6.7 \times 10^3 \le R_\theta \le 26.6 \times 10^3$. The last two sets of data were taken with hot wires and the first one with flattened pitot tubes.

The profiles in Fig. 1 are similar to those plotted by George and Castillo[13] and show the same Reynolds-number dependence. However, the profiles tend to approach an asymptotic profile as the Reynolds number increases. Thus, this behavior is consistent with the AIP discussed earlier. The figure mixes experimental data acquired by changing the downstream location with that obtained by fixing the streamwise position and varying the upstream conditions (such as the wind-tunnel speed). It has generally been the belief of the turbulence community that these should produce the same result. The boundary-layer thickness at $U/U_\infty = 0.99$ has been used as the outer length scale δ_{99} for all profiles shown in this paper. For the

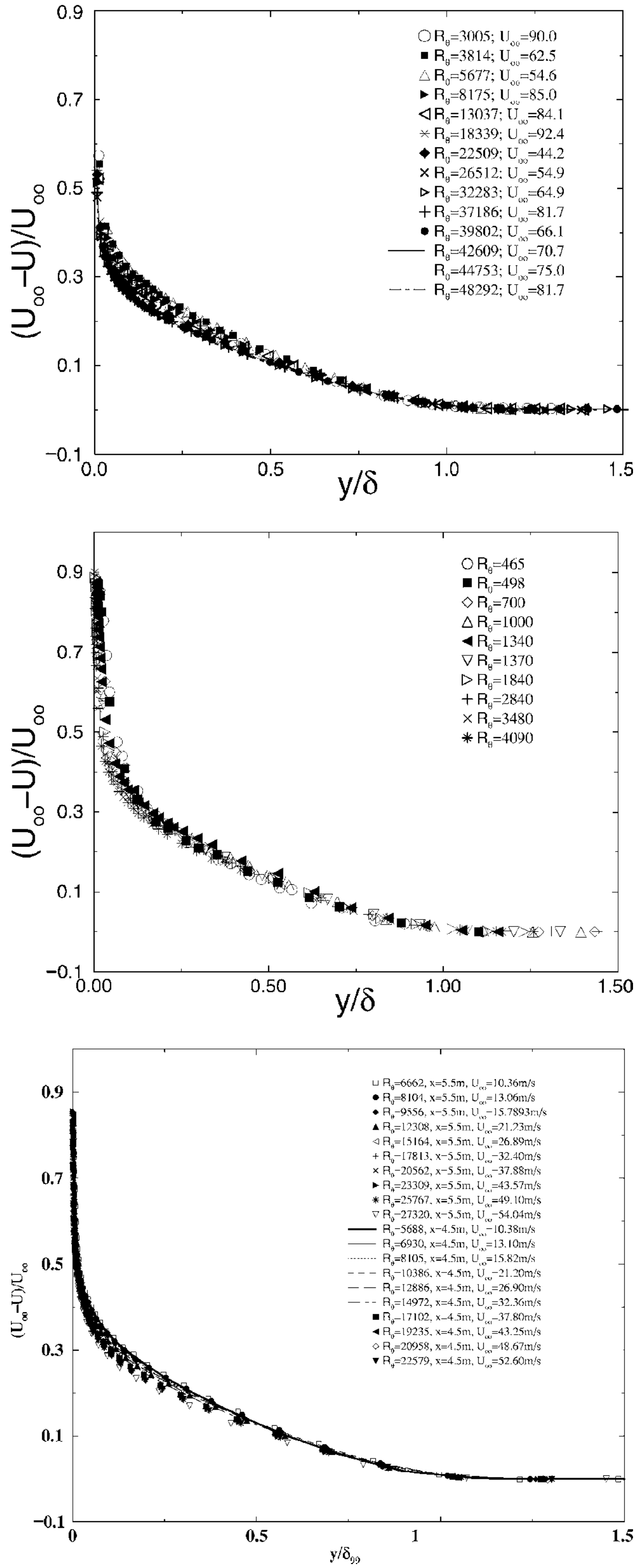

Fig. 1 Mean velocity deficit profiles in ZPG normalized by U_∞ and δ_{99} for various upstream conditions: top, Smith and Walker[17]; center, Purtell et al.[18]; and bottom, Österlund.[19]

Österlund data, the streamwise coordinate was fixed at $x = 5.5$ m from the leading edge, and only the wind-tunnel speed was varied (from about 10 m/s up to about 50 m/s).

These preceding results can be contrasted with the data of Wieghardt[20] and of Castillo and Johansson,[21] both which were taken by varying the distance downstream for fixed upstream conditions. Wieghardt kept the wind-tunnel speed at 33 m/s and varied only the downstream distance x. Castillo and Johansson measured at two wind-tunnel speeds (5 and 10 m/s) and fixed the size of the trip wire

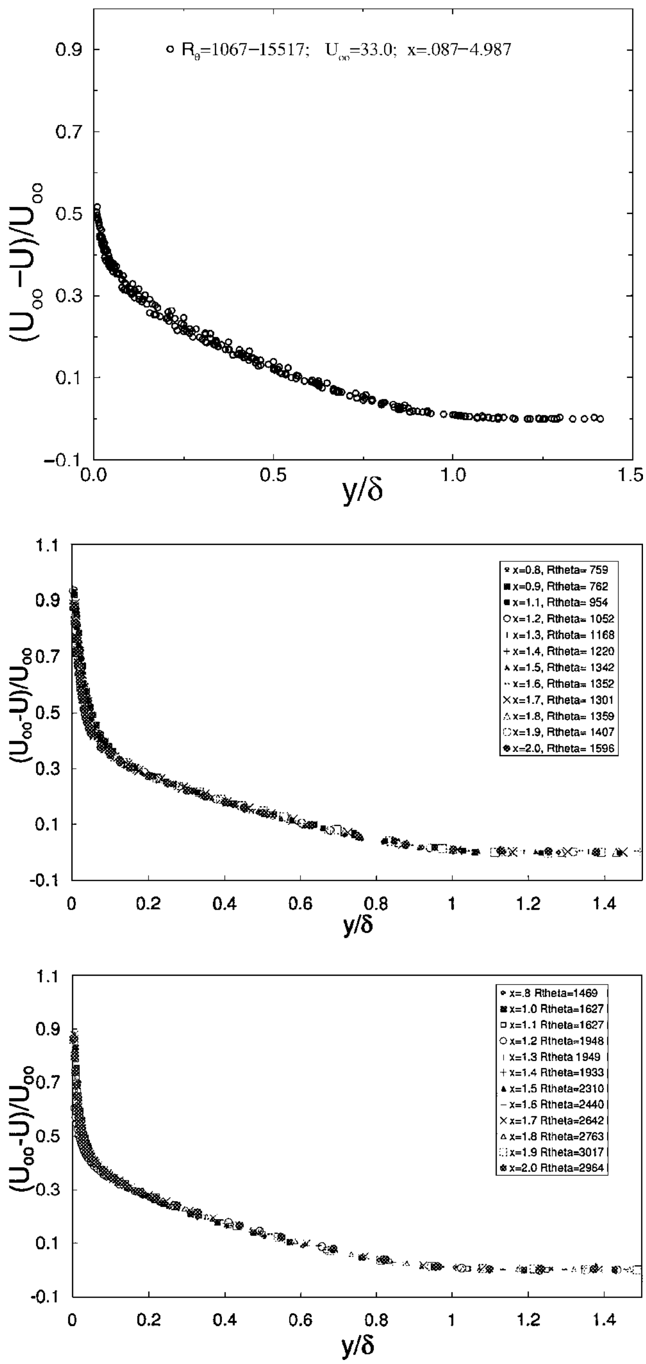

Fig. 2 Mean velocity deficit profiles in ZPG normalized by U_∞ and δ_{99} for fixed upstream conditions: top, Wieghardt[20]; center, Castillo and Johansson[21] at 5 m/s; and bottom, Castillo and Johansson[21] at 10 m/s.

(diameter $d_o = 2$ mm) and its location ($x_o = 15$ mm from the leading edge). Then for each set of fixed upstream conditions, they also measured the downstream evolution of the boundary layer. As shown in Fig. 2, the outer profiles for these data collapse with just U_∞ alone.

Thus, it is clear that the primary cause for the lack of collapse of the Smith and Walker, Purtell et al., and Österlund profiles was caused by the changes in the upstream conditions and not to the local Reynolds number. The data from Wieghardt cover a range of Reynolds number from low to moderately high (from about $10^2 \leq R_\theta \leq 15 \times 10^3$). It is within this range that most of the variation with Reynolds number has been observed, yet clearly the figure shows there is none. The data from Castillo and Johansson vary from about $1.3 \times 10^3 \leq R_\theta \leq 3 \times 10^3$, which is the range of the greatest variation seen by George and Castillo.[13] Clearly these data also collapse with just U_∞.

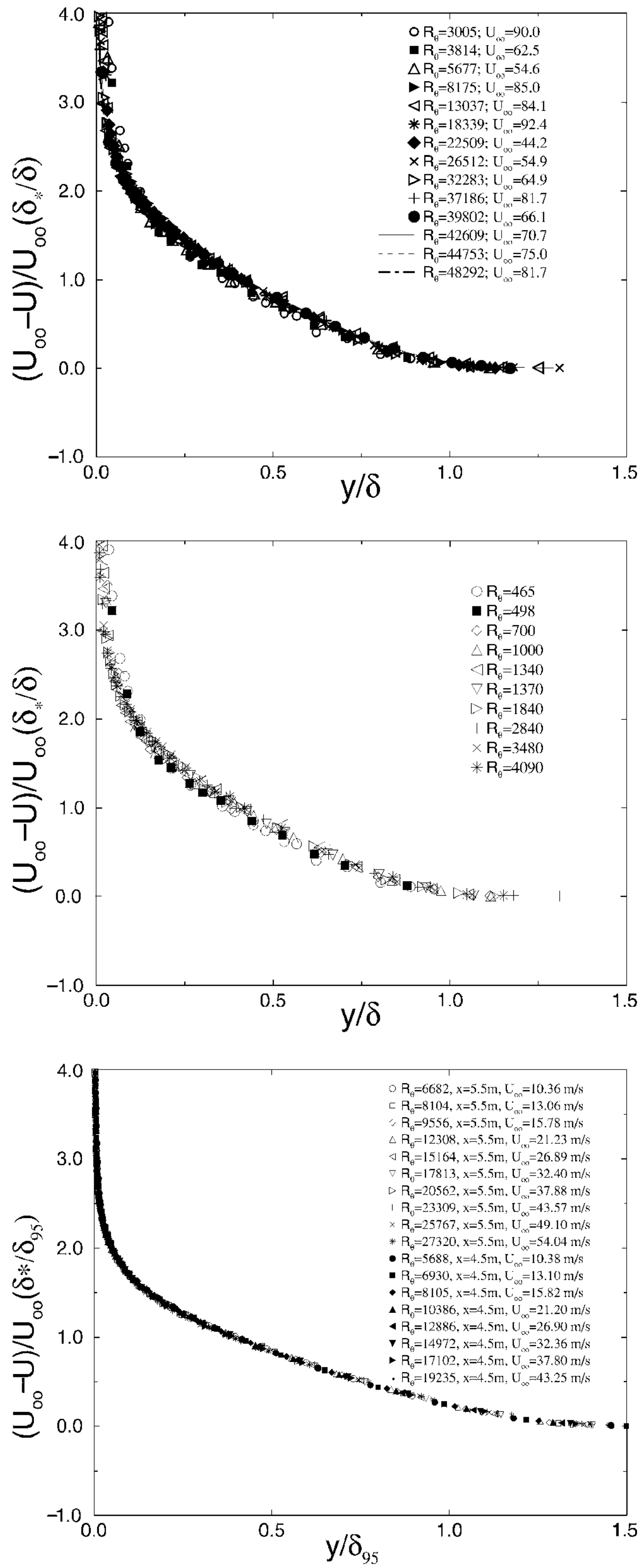

Fig. 3 Mean velocity deficit profiles in ZPG normalized by $U_{so} = U_\infty (\delta_*/\delta_{99})$ and δ_{99}: top, Smith and Walker[17]; center, Purtell et al.[18]; and bottom, Österlund.[19]

Obviously almost all of what was previously believed to be local Reynolds-number effects is caused instead by the upstream conditions. Exactly what it is about the upstream conditions that determines this behavior must be the subject of investigation, but it certainly would seem to involve some kind of upstream Reynolds number. On a practical level, it might also provide interesting opportunities for boundary-layer control.

The same data of Smith/Walker, Purtell et al., and Österlund are shown in Fig. 3 but normalized by the outer velocity scale

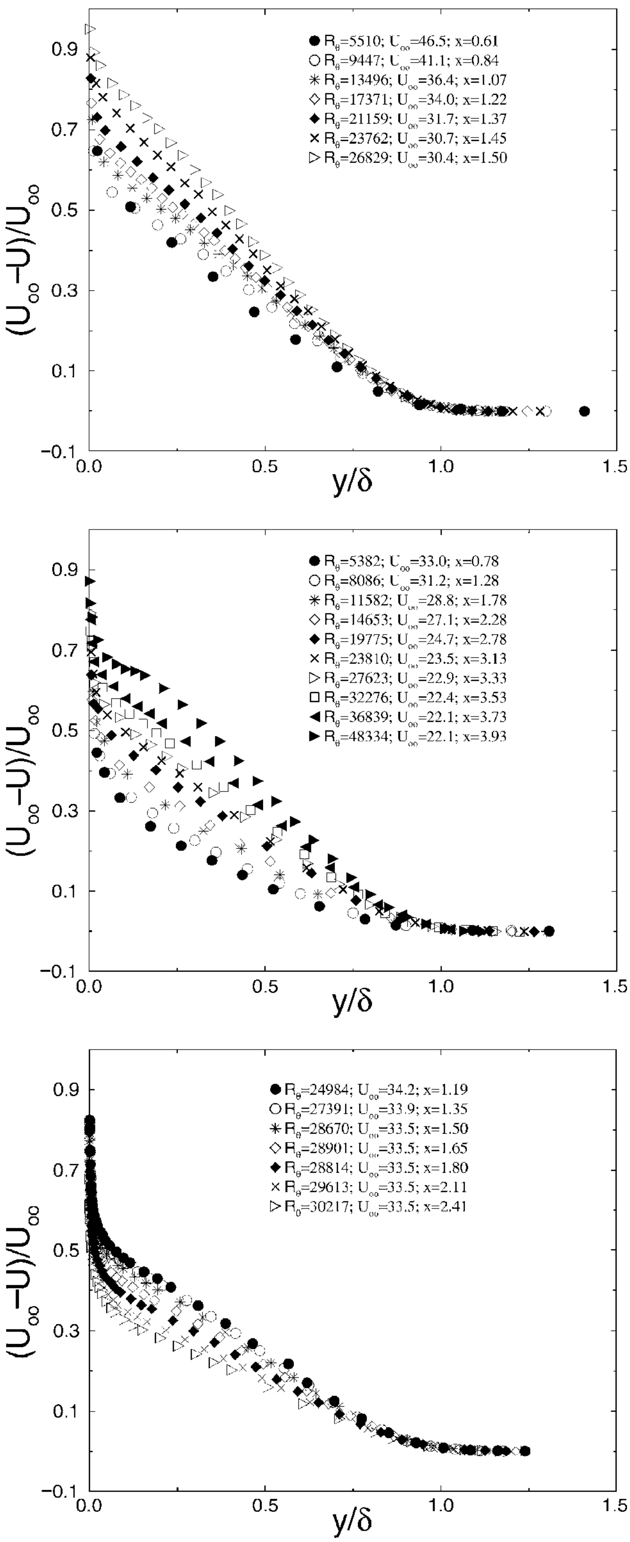

Fig. 4 Mean velocity deficit profiles in APG boundary layers normalized by U_∞ and δ_{99}: top, the strong APG data of Newman[22]; center, the strong APG data of Ludwieg and Tillmann[23]; and bottom, the "relax flow" data of Bradshaw and Ferriss.[24]

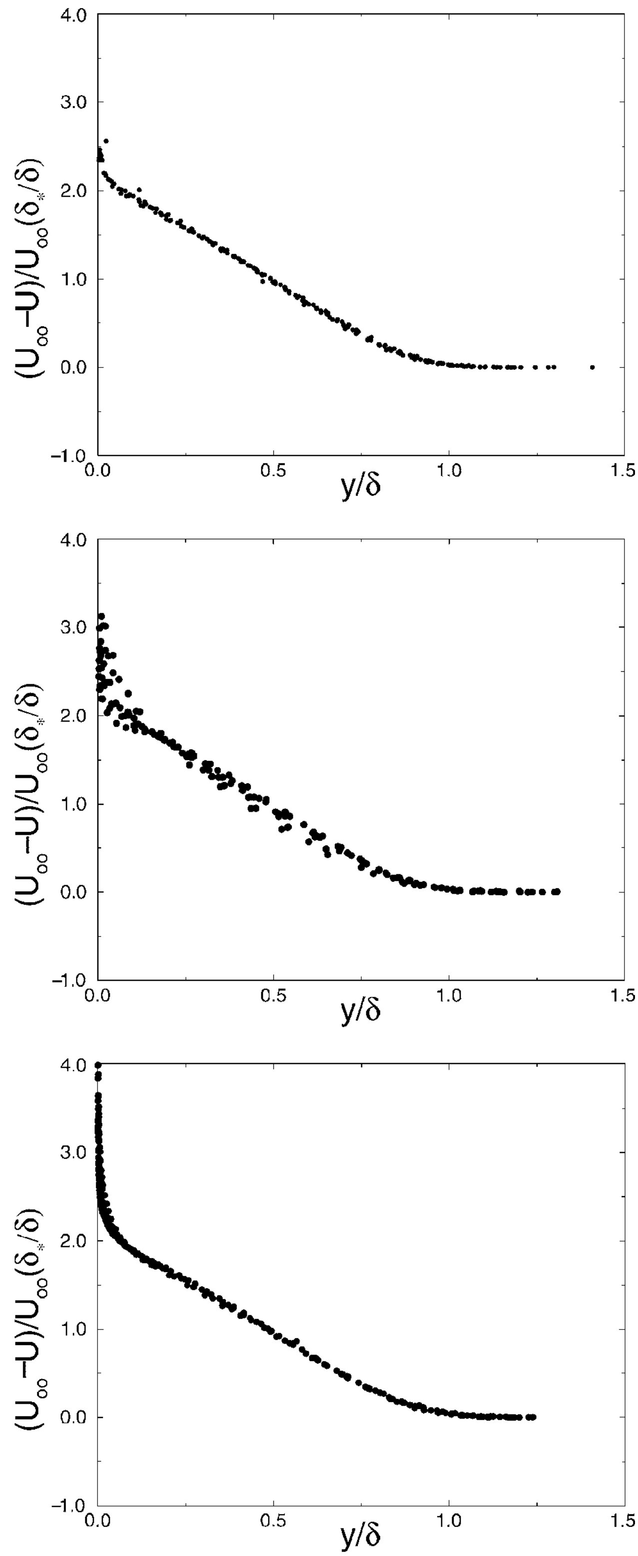

Fig. 5 Mean velocity deficit profiles in APG boundary layers normalized by U_∞ (δ_*/δ) and δ_{99}: top, the strong APG data of Newman[22]; center, the strong APG data of Ludwieg and Tillmann[23]; and bottom, the "relax flow" data of Bradshaw and Ferriss.[24]

$U_{so} = U_\infty(\delta_*/\delta)$ and the boundary-layer thickness δ_{99}. All profiles appear to collapse to a single curve. Thus, this scaling removes all of the Reynolds number and initial condition dependencies from the outer flow, exactly as suggested by the separability hypothesis of Sec. III.

B. APG Velocity Profiles

Figure 4 shows the APG experimental data of Newman,[22] Ludwieg and Tillmann,[23] and Bradshaw and Ferriss[24] normalized by U_∞ and δ_{99}. The data of Newman at the top have a strong APG over an airfoil, and the profiles eventually separate. The range of Reynolds number based on the momentum thickness for these data is between $5.51 \times 10^3 \leq R_\theta \leq 26.8 \times 10^3$. The Ludwieg and Tillmann data at the center of Fig. 4 have also a very strong APG and eventually separation takes place from the diverging channel. These data vary in Reynolds number from about $5.4 \times 10^3 \leq R_\theta \leq 48.3 \times 10^3$. In the experimental data of Bradshaw and Ferriss (bottom panel of Fig. 4), the conditions of the flow are suddenly changed from moderate APG to ZPG, the so-called "relax flow." These profiles vary in Reynolds number from about $8.6 \times 10^3 \le R_\theta \le 22.6 \times 10^3$. Clearly, the mean profiles show a dependence on Reynolds number, although they tend to approach an asymptotic state as the Reynolds number increases.

The same experimental data are shown in Fig. 5, but normalized by $U_{so} = U_\infty(\delta_*/\delta)$ and δ_{99}. Clearly, the Zagarola/Smits scaling successfully removes all of the Reynolds number and/or upstream condition dependence of the mean profiles, even for "relax flows" and for nearly separated boundary layers. These profiles do not collapse at all if the data are normalized by the classical scaling $(U_\infty - U)/u_*$, where u_* is the friction velocity.

By contrast with the APG boundary layers near separation, which do show a dependence on local Reynolds number, the profiles of Fig. 6 collapse with just U_∞. The top panel of Fig. 6 shows the mild APG experimental data of Bradshaw,[25] the moderate APG experimental data of Bradshaw,[25] and the moderate APG data of Clauser,[5] normalized by U_∞ and δ_{99}. The upstream conditions for each data set were kept nearly fixed, further emphasizing the importance of the upstream conditions and their effect on the flow downstream. Also, notice that the range of Reynolds number of these profiles is about the same as in preceding figure. Clearly the upstream condition effect dominates that of the local Reynolds number for all but the most extreme pressure gradients.

C. FPG Velocity Profiles

The mild favorable pressure gradient data of Herring and Norbury[26] and the moderate FPG data of Ludwieg and Tillmann[23] normalized by U_∞ and δ_{99} are shown in Fig. 7. Herring and Norbury[26] covers a range of Reynolds numbers from $3392 \le R_\theta \le 4289$. The data from Ludwieg and Tillman has a range from $1007 \le R_\theta \le 4061$. These profiles collapse reasonably well with just U_∞ because the upstream conditions are fixed. Thus, the Z/S scaling is not necessary.

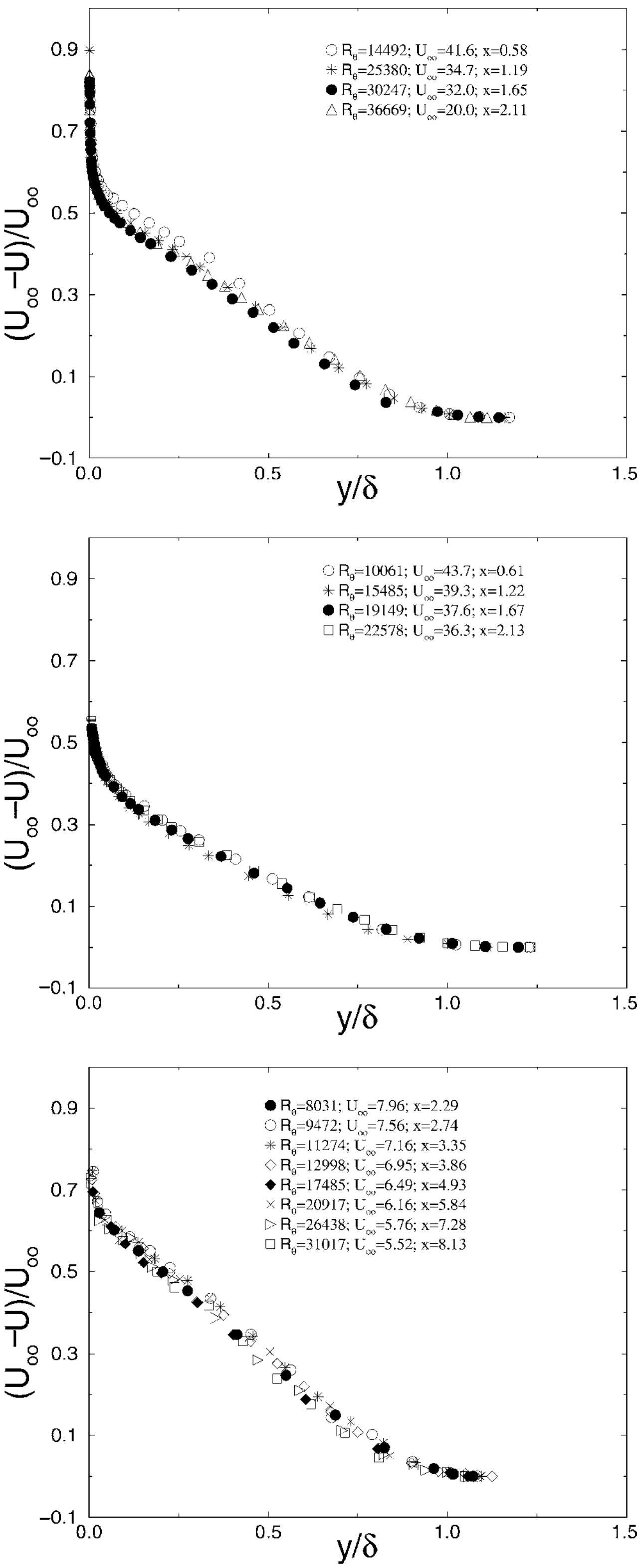

Fig. 6 Mean velocity deficit profiles in APG boundary layers normalized by U_∞ and δ_{99}: top, the moderate APG data of Bradshaw[25]; center, the mild APG data of Bradshaw[25]; and bottom, the moderate APG data of Clauser.[5]

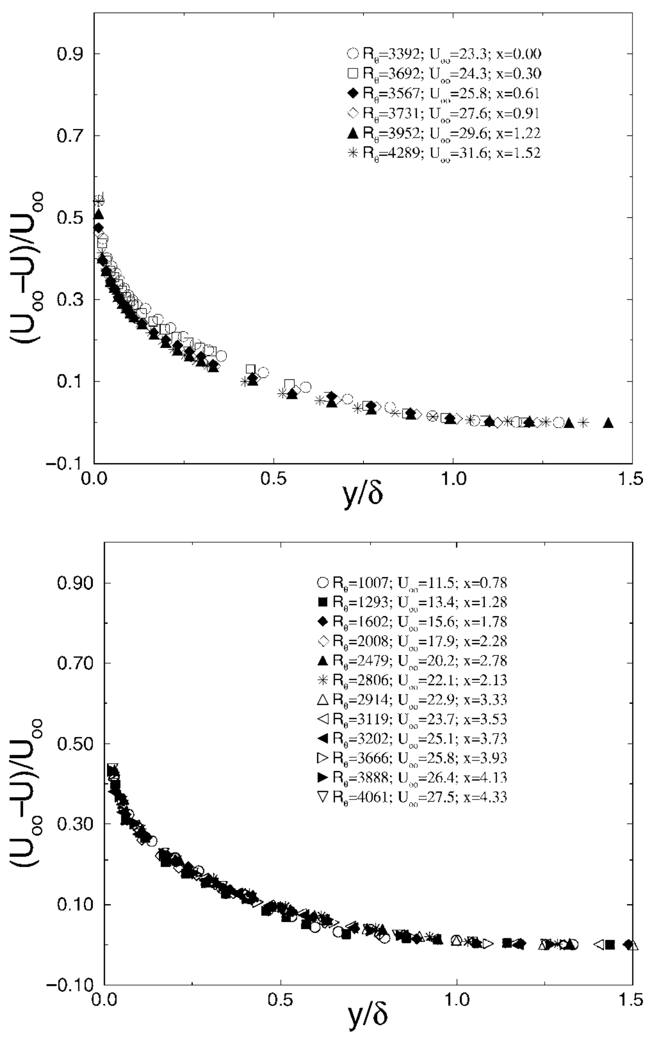

Fig. 7 Mean velocity deficit profiles in FPG boundary layers normalized by U_∞ and δ_{99}: top, the data of Herring and Norbury[26] at mild FPG; and bottom, the Ludweig and Tillman[23] data at moderate FPG.

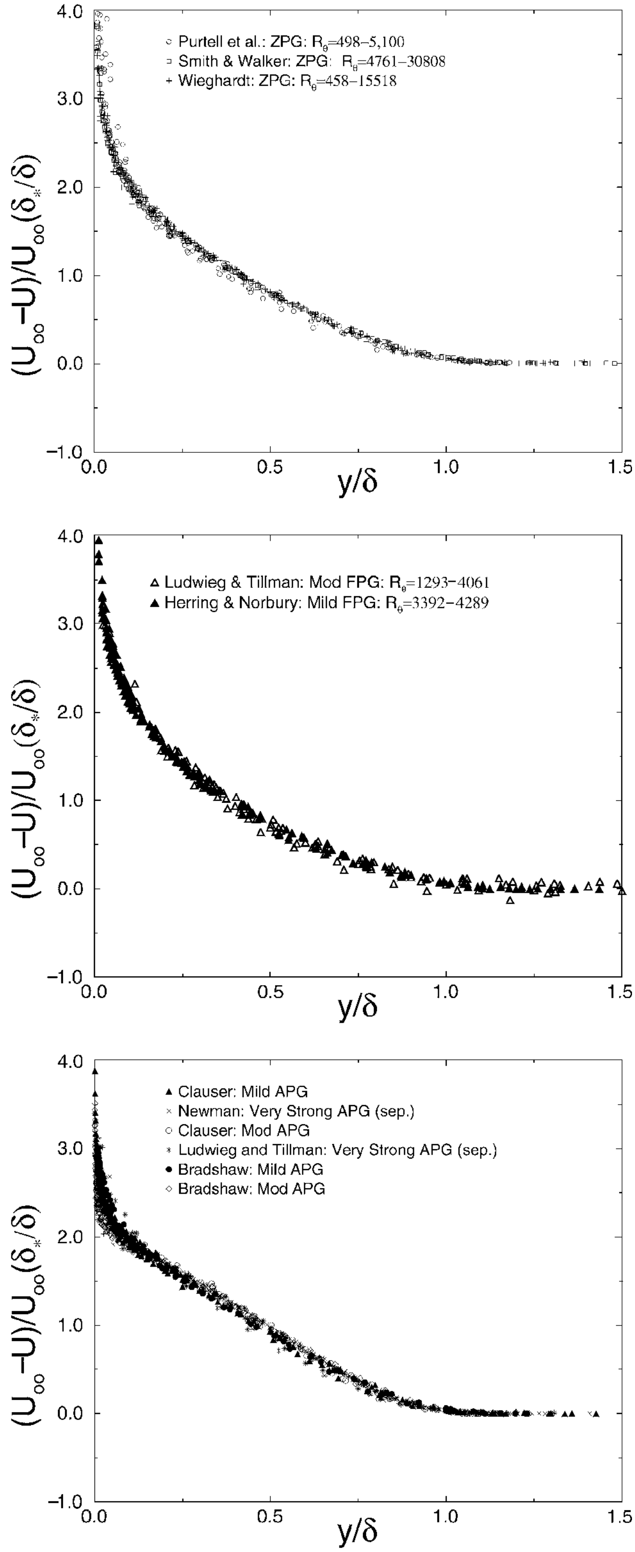

Fig. 8 Three basic velocity deficit profiles in turbulent boundary layers normalized by U_∞ (δ_*/δ_{99}) and δ_{99}: top, ZPG profiles; center, FPG profiles; and bottom, APG profiles.

D. Three Mean Profiles

The purpose of this section is to use the Z/S scaling to show that there are only three basic profiles needed to characterize turbulent boundary layers: one each for ZPG, APG, and FPG. Moreover, this result is independent of the strength (weak, mild, strong) of the pressure gradient. These correpond to the observation of Castillo and George[2] that there are only three values of Λ, hence only three possible solutions to the RANS equations for the mean velocity deficit profile.

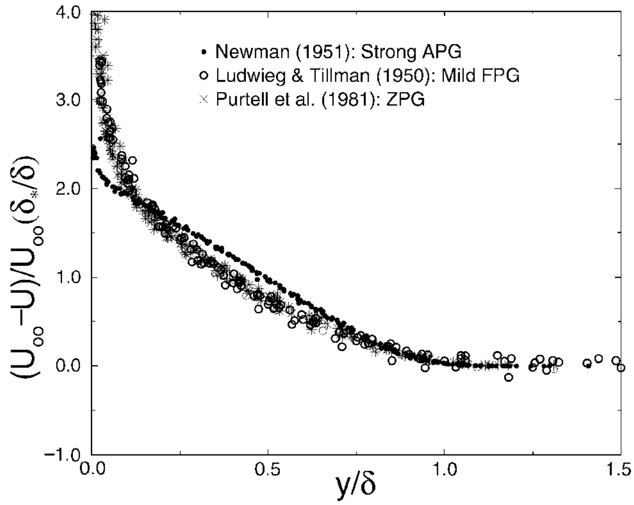

Fig. 9 Three basic velocity deficit profiles in turbulent boundary layers normalized by U_∞ (δ_*/δ_{99}) and δ_{99}.

The first plot from the top of Fig. 8 shows some of the ZPG data of Figs. 1 and 3. Also note that all ZPG data collapse to a single curve when normalized by $U_{so} = U_\infty(\delta_*/\delta)$. Because δ_*/δ goes to constant in the GC theory, it is this profile that represents their limiting infinite Reynolds-number profile.

The second plot from the top of Fig. 8 is the FPG data, also plotted using $U_{so} = U_\infty\delta_*/\delta$. Even though these profiles collapsed using U_∞ only for fixed upstream conditions, they might collapse to different curves for different conditions. The Z/S scaling clearly collapses all of the FPG data to one single curve, consistent with the separability hypothesis of Eq. (12).

The third plot from Fig. 8 represents some of the APG data already discussed along with some of the profiles from the "relax flow" data of Bradshaw and Ferriss. Again, it is clear from Fig. 8 that all APG do collapse to a single curve when normalized using the Z/S scaling. As for the ZPG and FPG, these profiles represent the asymptotic profiles from the George and Castillo theory, and all of the effect of the upstream conditions (and local Reynolds number) is confined to the ratio δ_*/δ. Because this ratio is asymptotically constant, the constant must reflect the initial conditions.

Figure 9 shows the ZPG, APG, and FPG data plotted together using the Z/S scaling. It illustrates that there are indeed only three basic profiles in turbulent boundary layers. The APG profile is distinctly different from the other two, which are nearly the same.

V. Conclusions

It is clear that the way boundary layers are generated plays an important role in the downstream flow development. Also, the Reynolds-number dependence observed in some of the outer velocity deficit profiles is primarily caused by the changes in the upstream conditions and not to the local Reynolds number.

The empirical velocity scale determined for zero-pressure-gradient turbulent boundary layers by Zagarola and Smits,[1] $U_\infty(\delta_*/\delta)$, was derived using similarity principles. This scaling is successful in removing both the local Reynolds-number dependence of the outer mean velocity profiles as well as the effects of the upstream conditions. Application of this scaling produces only three profiles, perhaps for all turbulent boundary layers, regardless of the strength of the pressure gradient: one for adverse pressure gradient, one for favorable pressure gradient, and one for zero pressure gradient.

These results were anticipated by Castillo and George,[2] who argued from similarity grounds that only three such profiles were possible. In particular, they showed from the experimental data that there appeared to be only three values of the pressure parameter Λ: one for ZPG, $\Lambda = 0$, one for FPG, $\Lambda = -1.915$ and, one for APG, $\Lambda = 0.22$. Furthermore, they argued that because the outer boundary layer equation is only dependent on this pressure parameter, then there can be only three corresponding profiles. Remarkably, the Z/S scaling has revealed them.

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